# Vertical Reference Flight Trajectory Optimization with the Particle Swarm Optimisation

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**3 authors:**

Alejandro Murrieta Mendoza
Amsterdam University of Applied Sciences/Centre for Applied Research on Education


Hugo Ruiz
Laboratoire d'Informatique, de Robotique et de Microélectronique de Montpellier (LI…


Ruxandra Mihaela Botez
École de Technologie Supérieure

# VERTICAL REFERENCE FLIGHT TRAJECTORY OPTIMIZATION WITH THE PARTICLE SWARM OPTIMISATION

Alejandro Murrieta-Mendoza, Hugo Ruiz, Ruxandra Mihaela Botez
The Research Laboratory of Active Control, Avionics, and Servoelasticity (LARCASE) - Université du Québec – École de Technologie Supérieure
1100 Rue Notre-Dame Ouest, Montreal, QC H3C 1K3, Canada
ruxandra.botez@etsmtl.ca*, alejandro.murrieta-mendoza.1@etsmtl.net

**ABSTRACT**
The consumption of fossil fuels in order to power flights leads to undesirable pollution particles to be released to the atmosphere. Fuel also represents an important expense for airlines. For these reasons, it is of interest to reduce fuel burn for a given flight. In this article, the altitudes followed by a commercial aircraft during the cruise phase of a flight, also called vertical reference trajectory, were optimized in terms of fuel burn. The airspace was modelled under the form of a unidirectional graph. Fuel burn was computed using a numerical performance model. The weather forecast was obtained from the model delivered by Environment Canada. The selection of waypoints where to execute the changes in altitudes that provided the most economical flight cost in terms of fuel burn was determined using the particle swarm optimisation (PSO) algorithm. The trajectories provided by the algorithm developed in this paper were compared against simple geodesic trajectories to validate its optimization potential, and against as flown trajectories. Results have showed that up to 6.5% of fuel burn can be saved comparing against simple trajectories, and up to 3.1% was optimized comparing against as flown trajectories.

**KEY WORDS**
Trajectory Optimization, Fuel Burn, Pollution, Carbon Dioxide, Commercial Aircraft, Step Climb, Altitude, Navigation.

## 1. Introduction

The aeronautical industry is aware of their pollution share as it has been estimated that 2% of the total $CO_2$ liberated in the world are attributed to aeronautical activities [1]. A recent study has pointed out that $CO_2$ may alter the wind currents making a transatlantic city pair flight longer requiring more fuel [2]. Another study has shown that operations such as the missed approach procedure also demand important quantities of fuel releasing high amounts of pollution to the atmosphere [3, 4].

Since engines require important quantities of fuel to power flights, it is not an easy task to reduce fuel emissions. However, every generation of aircraft and every generation of engines are more efficient diminishing drag and producing thrust requiring less fuel. New materials have helped reduce the aircraft weight lowering the fuel requirements. All these advances although extremely desirable, require the expensive tasks of development of new aircraft, or to modify the structure of current ones. Besides, comparing against other industries aeronautical technology is hard to be certified; therefore, any improvement would require longer time in order to be tested and validated to be available in the market. Furthermore, new technologies sometimes apply only to new aircraft. Thus, new technology is implemented by airlines until an aircraft has reached the end of its lifecycle and it requires to be replaced by a new one.

Another way reducing fuel burn is by modifying operations. This idea was discussed in [5] where simple operations such as using energy from the gate instead of using the Auxiliary Power Unit (APU) are desirable. Another simple way to reduce fuel consumption is by washing engines, by performing the taxi procedure with one engine or assisted with ground equipment. A full discussion regarding the different operation stages and the different opportunities to reduce fuel burn and current measures being implanted were discussed in detail in [6].

Optimization of the cruise phase operations are an interesting alternative to reduce fuel burn. Proposing more efficient trajectories in terms of fuel burn during the cruise phase is seen as a way to reduce fuel consumption. Airlines have dispatch teams that plan the most economical trajectories taking into account different constraints such as no fly zones, airways, and weather constraints. The trajectory delivered for these teams consist in the ground track to follow along with the altitudes and speeds that the aircraft should attain at each waypoint. This is also known as the vertical reference trajectory. The flight delivered by these teams is loaded by the crew to the Flight Management System (FMS). Normally this device is loaded with a function able to optimize in real time the reference trajectory in terms of flight cost [7].

Although trajectory optimization is a way to reduce fuel consumption, there is an opportunity to improve the reference trajectory optimization algorithms in altitude and speed [8, 9]. Many different algorithms have been proposed to optimise the trajectory.

Dynamic programming was implemented in [10] with an iterative process where the trajectory of reference moved around the search space converging to the optimal trajectory. Dynamic programming with neural networks was also implemented in [11]. The Floyd-Warshall algorithm was studied in [12].

Optimal control was implemented following current air traffic management constraints such as constant speed and constant segment altitudes [13]. Also using optimal control, the trajectory was optimized while avoiding contrails formation zones [14]. Optimal control was also used to compute the most economical speed during the cruise phase [15].

Flights were optimized in the vertical reference trajectory by using the golden section search to select the most economical cruise altitude during short flights [16]. Algorithms aiming to reduce the search space to improve the aircraft's convergence as in [17]. Beam search, a variation of branch and cut, was suggested in [18], and it was later improved to reduce the computation time [19] finding the global optimal solution. An algorithm able to find the optimal altitude for a fixed altitude cruise and a fuel burn estimator was studied in [20].

Genetic algorithms were used to optimize the trajectory optimization in the lateral reference trajectory [21], in the vertical reference trajectory [22], and in simultaneously optimizing the lateral and the vertical dimensions and the Mach number [23].

In [24] the reference trajectory during the cruise phase was optimized first in the lateral reference trajectory, and then, following, the waypoints locations were to execute step climbs in order to provide the most economical flight in terms of fuel burn were found.

The Dijkstra's algorithm was implemented for general aircraft to avoid obstacles [25]. The Dijkstra's algorithm was also used to look for winds to reduce the flight cost during cruise [26]. A*, a variation of Dijsktra's algorithm was used in [27] to find the optimal trajectory while avoiding objects representing weather constraints .

Metaheuristic algorithms have also been implemented in the trajectory optimization problem such as the artificial bee's colony as in [28] where the most economical vertical reference trajectory with a time constraint was found. The Ant colony optimization was implemented in order to find the most economical lateral reference trajectory taking advantage of winds [29]. The ant colony optimization was also implemented to find the most economical Mach numbers that fulfills a time constraint [30]. Taboo search was implemented to optimize the trajectory focusing its implementation in future Flight Management Systems [31].

In this paper, the first objective is to develop an optimization algorithm able to find the combination of altitudes that provide the most economical flight in terms of fuel burn. The flight is only optimized for the cruise phase as in long haul flights it is the phase that burns the most fuel. The second objective of this paper is to observe the optimization potential to reduce the fuel burn in the vertical dimension by the Particle Swarm Optimization (PSO).

The optimization algorithm used in this paper was the PSO. This metaheuristic algorithm tends to behave well in large search spaces converging to the optimal solution [32].

This paper is organized as follows. Firstly, the aircraft's fuel burn model is described. Secondly, the fuel burn computation is discussed. Thirdly, the weather forecast model is presented. Then, the search space is explained. Next, the PSO general theory is described followed by the description of its implementation for the trajectory optimization problem. Finally, results and conclusions are presented.

## 2. The Aircraft, Weather, and Search Space Models and the Fuel Burn Computation

The fuel burn model was obtained from experimental data. However, numerical performance can be as well created using flight simulators [33] or/and Aircraft Flight Manuals (AFM) [34].

A numerical performance model consists in sampled fuel burn (and horizontal traveled distance) for different flight phases at different flight conditions. Since this work is directly related to the cruise phase, only data for the fixed cruise phase, climb with a Mach number and descent with a Mach number are of interest.

The numerical performance model can be seen as a black box, where flight information such as Altitude (ft.), Mach number, Aircraft's gross weight (kg), and The ISA temperature deviation (ISA_DEV) are introduced. As a result, fuel flow, or fuel burn and horizontal traveled distance are provided. This can be seen in Figure 1.

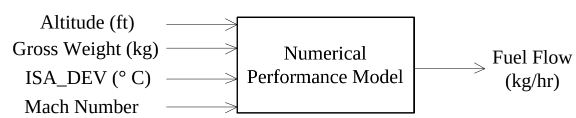


Figure 1. The performance model seen as a black box. All inputs should be available in order to obtain the output.

### 2.1 Fuel Burn

The aircraft's weight diminishes as fuel is burned. For this reason, the values are not exactly the ones available in the numerical performance model. The same happens for ISA temperature deviation (*ISA_DEV*) as temperature changes all over the flight. For this reason, interpolations are required to compute the exact fuel flow or the exact fuel burn and horizontal traveled distance. This was detailed in [35, 36] and a brief explanation is provided next.

The trajectory is divided in equidistant segments in order to compute the flight cost. Segments are separated by 20 nautical miles to achieve a good compromise between accuracy and computation time. At the beginning of each segment the next operations are performed.

a. The fuel burn burned required to travel the previous segment is subtracted from the aircraft's gross weight.
b. Weather information such as wind speed, wind direction, and temperature are used to compute the Ground Speed (GS) – Aircraft's speed respect to the ground- and the ISA_DEV.
c. Linear interpolations are executed to compute the fuel flow or the fuel burn and the horizontal traveled distance.

The fuel burn is computed in two different ways depending on the cruise regime: steady altitude or change of altitude.

Fixed altitude cruise:
The information delivered by the numerical performance model is given under the form of Fuel Flow (kg/hr). *Flight Time* is then required to compute the *Fuel Burn* for a given segment. Knowing the aircraft's *GS* for a 20 nm segment, *Flight Time* can be easily computed. With this information, the Fuel burn for a given segment can be computed with Eq (1):

$$Fuel\ Burn(kg) = Fuel\ Flow\left(\frac{kg}{hr}\right) \times Flight\,Time(hr) \qquad (1)$$

Change of altitude:
When a change of altitude is required, the numerical performance model provides the fuel burn to climb from the initial altitude to the objective altitude, as well as the required horizontal traveled distance. The rest of the segment is computed as a fixed altitude cruise as in the next steps:

a. The fuel burn burned required to travel the previous segment is subtracted from the aircraft's gross weight.
b. The fuel burn required to change altitude (climb or descent) to the targeted altitude and the required horizontal distance are computed.
c. The fuel burn required to change altitude is subtracted from the aircraft's gross weight.
d. The required horizontal traveled distance is reduced from the segment distance (e.g, distance = 20 nm – horizontal traveled distance).
e. The remaining distance is computed as a fixed altitude

Fuel Burn Computation
The total fuel burn is then the sum of all the segments taking into account the change of altitude cost.

$$Total\ Fuel\ Burn = \sum_{j}^{n} (Altitude\ Change\ Fuel\ Burn + Cruise\ Fuel\ Burn \qquad (2)$$

Evidently, *Altitude Change Fuel Burn* = 0 when there is no change of altitude in the segment *j*.

## 2.2. Weather Model

Weather plays an important role in the flight cost. Temperature influences the engine's performance. Headwinds make the flight longer as it *pushes* the aircraft back incrementing the flight time, on the other hand, tailwinds *pull* the aircraft forward reducing the flight time.

The weather forecast used for the algorithm was downloaded from the open source provided by Weather Canada. This information is provided under the form of a 0.6 km x 0.6 km grid, at different pressures (Flight Level) at 3 hours block from 0h to up to 144h forward. At each grid vertex, information such as wind speed, wind direction and temperature are available

Similarly, as with the numerical performance model, because of the way data is provided, the aircraft is rarely at the vertexes. For this reason, interpolations in time, flight level, and within the four vertexes enclosing the aircraft are required. The interpolation order is shown in Figure 2.

## 2.3. Search Space

The search space was modeled under the form of a unidirectional graph where the vertexes are represented by the waypoints and the weights are the fuel burn required to connect two consecutive vertexes.

### 2.3.1 Boundaries

The search space is defined by two different boundaries. The first boundary is the flight distance defined by the Top of Climb (ToC) and by the Top of Descent (ToD) geographical positions. The second boundary consists in the minimal and the maximal cruise altitude. The search space is graphically described in Figure 3.

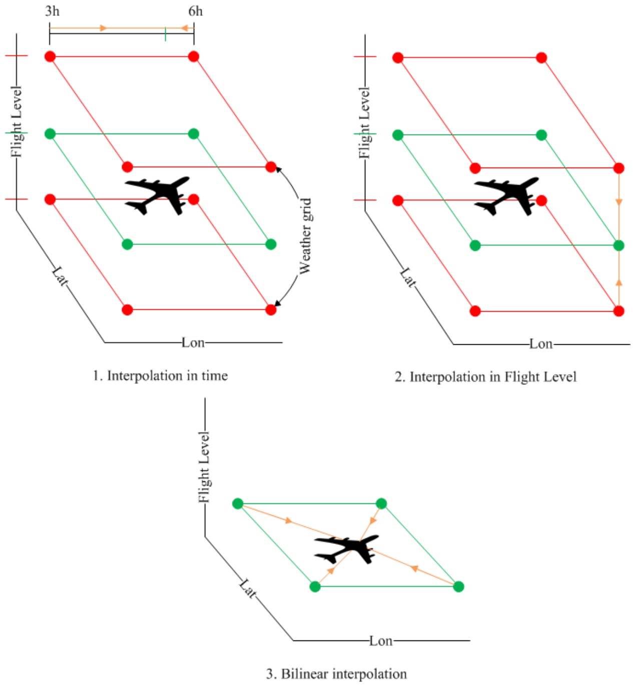

Figure 2. Interpolations to determine the weather conditions at the exact aircraft's position.

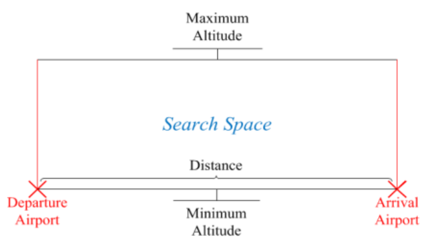

Figure 3. The search space defined by the flight distance and by the maximal and minimal cruise altitudes

#### 2.3.2 The ground track waypoint placement

As the algorithm is able to change altitude only when the aircraft is located at a waypoint, the number of waypoints placed in the search space to crate the graph has an impact on the algorithm performance. If many waypoints are placed for given flight, the search space is more precise; however, more combinations will be available. This might lead to increment on the computation time. A good compromise was to select 34 waypoints for a long haul flight. This is also the typical number of waypoints in a long haul flight. The waypoints separation can be seen in Figure 4.

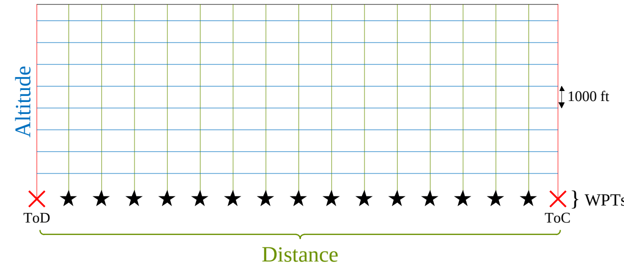

Figure 4. Search Space for the cruise phase.

However, due to traffic restrictions, flights going westbound fly at odd altitudes, while flights going eastbound fly at even altitudes. For this reason, when changing altitudes step climbs of 2,000 ft are executed. This allows the aircraft to finish the flight at any altitude. The search space for a given flight taking into account the step climb restriction can be seen as in Figure 5. Note that the aircraft can end the cruise phase at any of the red crosses in the red side.

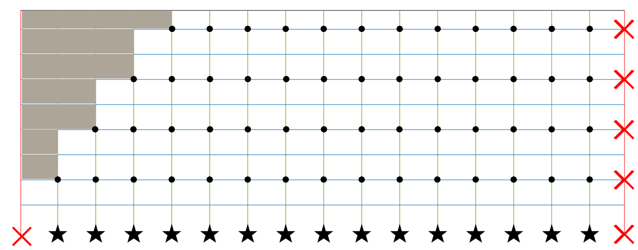
Figure 5. Search space for 2,000 ft step climb at different waypoints

## 3 The Optimization Algorithm for the Vertical Reference Trajectory

### 3.1 The Typical Particle Swarm Optimization Algorithm

The Particle Swarm Optimization (PSO) algorithm emulates the social behaviour of animals such as the mobility of bird flock or fish school. This algorithm differs from other social algorithms such as ant colony or bee colony because ant and bees, in their societies, they don't have any leader guiding the research, they use the intelligence of the community. The PSO on the other had has a leader that the community is following. However, this leader can change at any time such as the bird flock leader rotation.

The standard particle swarm algorithm operates by iteratively searching within a region in which three parameters influence the actual position of the particle: the position of the best trajectory for all particles, the position of the most promising trajectory from a cluster of particles and the displacement velocity of each particle. The PSO behaviour can be described with Eq. (3) and Eq. (4).

$$D_i(k+1) = \omega R_\omega D_i(k) + c_1 R_1\big(B_l - X_i(k)\big) + c_2 R_2\big(B_g - X_i(t)\big) \quad (3)$$

$$X_i(k+1) = X_i(k) + D_i(k+1) \quad (4)$$

Where $k$ is the current iteration, $D$ is the particle's displacement, $X$ is the actual particle position. $B_l$ represents the best local trajectory $B_g$ is the best global trajectory. The inertia weight ($\omega$) is a parameter that has an influence on the next iteration and represents the particle opposition to displacement. Parameters $R_w$, $R_1$, and $R_2$ are random vectors with values (0,1) which weight the influence of each parameter in the equation, finally $c_1$ and $c_2$ are positive constants that represent the influence of the local and the global particle for the current displacement $D_i$.

### 3.2 The Swarm Optimization Algorithm for the Vertical Reference Trajectory

For the vertical reference trajectory problem, the actual position $X$ corresponds to the aircraft's altitude, $D$ corresponds to the desired change of altitude, the best trajectory for all particles ($B_g$) is the current most economical trajectory, and the local best ($B_l$) is the current most economical trajectory from a cluster of trajectories. Each waypoint is mapped to a displacement $D$ as in Eq (5)

$$D_i = [d_i^1, d_i^2, \dots, d_i^n] \quad (5)$$

The position $X$ is mapped for each waypoint as in Eq (6)

$$X_i = [x_i^1, x_i^2, \dots, x_i^n] \quad (6)$$

As constraints, the displacement $D$ can't be higher than 4,000 ft (or 2 consecutive step climbs.), or a single descent of 2,000 ft.

Referring back to Eq. (3), there are three factors that influence the change of altitude in a trajectory:

- The inertia of the previous movement ($\omega$). This value
- The influence of the local best ($B_l$)
- The influence of the global best ($B_g$)

After computing $X$, it might happen that the trajectory is not valid. A verification is required.

**Altitude Verification**

Displacements can generate altitudes such as the ones in Eq (7). These altitudes are such as 36,602.24 ft. These altitudes are not valid due to air traffic control management. These altitudes are then identified and rounded to the closest 1,000 ft value.

$$\begin{pmatrix} 34000 \\ 34000 \\ 34000 \\ 36602.24 \\ 34937.24 \\ 35398.2 \\ \vdots \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ \vdots \end{pmatrix} \quad (7)$$

Then, the waypoints' altitudes are studied trying to identify discrepancies such as the ones in Figure 6:

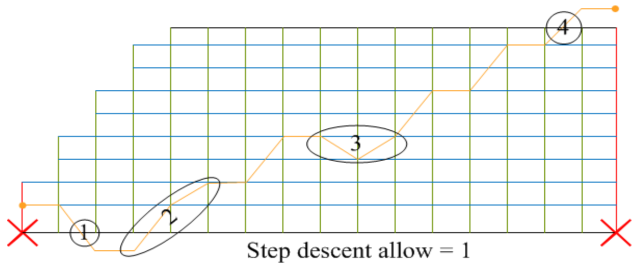


Figure 6. Discrepancies that may be found after the displacement of a trajectory

These discrepancies are described below:

1. Step descent over minimum altitude
2. 2 altitude changes consecutives.
3. Step descent and immediately after a step climb.
4. Step climb over the maximum altitude

The trajectory is then fixed, for example the example trajectory can be fixed to become the trajectory shown in Figure 7.

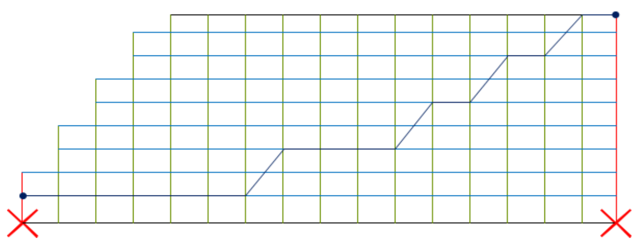

Figure 7. Corrected trajectory

### 3.3 The Developed Algorithm

<u>Step1</u>

Random trajectories are generated in 25 trajectories clusters. These trajectories connect the ToC and the ToD. These trajectories can either be at fixed altitudes or contain multiple altitudes changes. These trajectories are evaluated, thus the $B_l$ and the $B_g$ can be identified. A set of different random trajectories is shown in Figure 8.

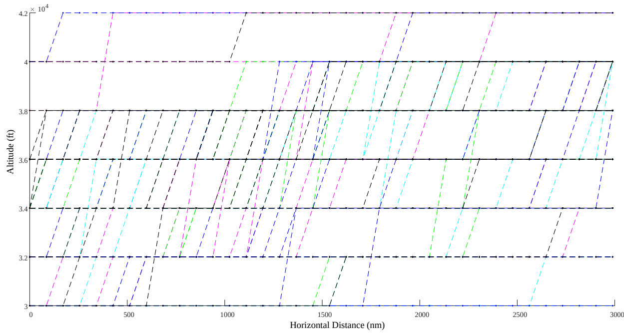

Figure 8. Random Generated Trajectories

Step 2:
Random vectors $R_w$, $R_1$ and $R_2$ are generated.

Step 3:
The displacement $D(k+1)$ for each trajectory in every cluster are computed.

Step 4
The position $X(k+1)$ is updated. At this moment the *Altitude Verification* process is performed

Step 5
The cost of the generated trajectories is computed. The new generated trajectory is kept if it is more economical than the older trajectory, otherwise it is discarded.

Step 6
The new local best and the new global best are identified. If the current local best within a given cluster is more expensive than any of the new generated trajectories, then the most economical trajectory becomes the new local best. If any of the local best trajectories are more economical than the current global best, then the most economical trajectory becomes the global best. If the total number of iterations is reached, the algorithms ends, otherwise Step 2 is executed.

The algorithm function, as well as the PSO function is graphically shown in Figure 9.

### 4.1 Local Cluster Trajectory Convergence

On Figure 10, a limited number of local trajectories and the *first global* best are shown. Note that there are many trajectories exploring the search space.

Once the iterations have been executed, the evolution of trajectories in Figure 10 is shown in Figure 11.

Here it can be seen that displacements $D$ moved the trajectories seeking convergence to the optimal trajectory. Note that the global trajectory changed from the one in Figure 10. This is because as the algorithms is executed, a better trajectory was found due to the particles' displacements.

### 4.2. The Optimization Potential Comparing Against a Geodesic Trajectory

For these tests the trajectory provided by the optimization algorithm are compared against the geodesic trajectory at 32.000 ft. As for coefficients, $\omega = 1$, $c_1 = 0.75$ and $c_2 = 0.75$.

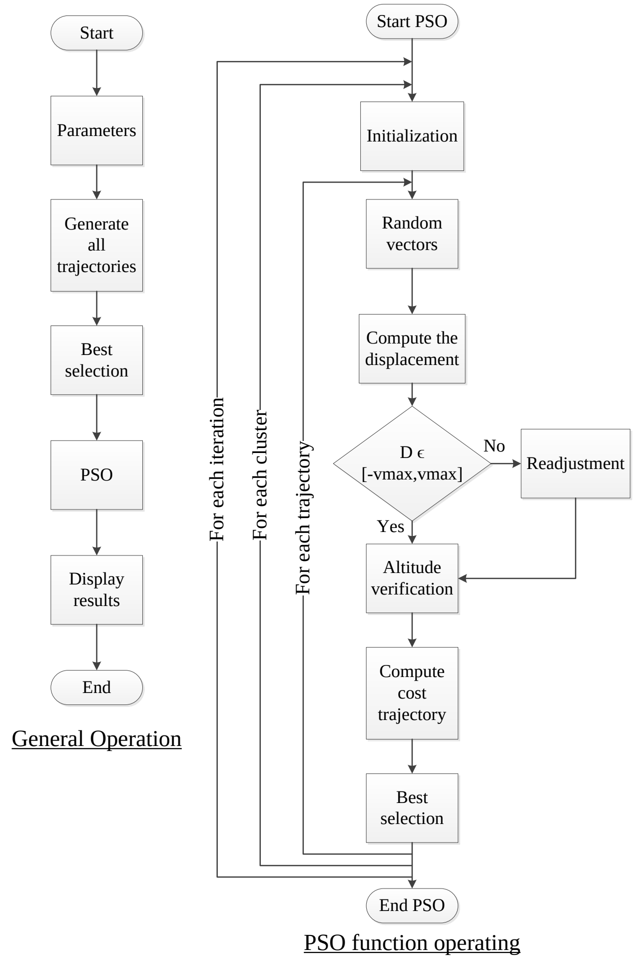

Figure 9. The algorithm and the PSO flowchart

The fuel burn difference between the flight provided from the algorithm and the fixed altitude geodesic for the flights Montreal (YUL) – Paris (CDG), Toronto (YYZ) – London (LDH), and Montreal to Vienna (VNN), and Vancouver (YVR) to Punta Delgada (LPA) are shown in Figure 12.

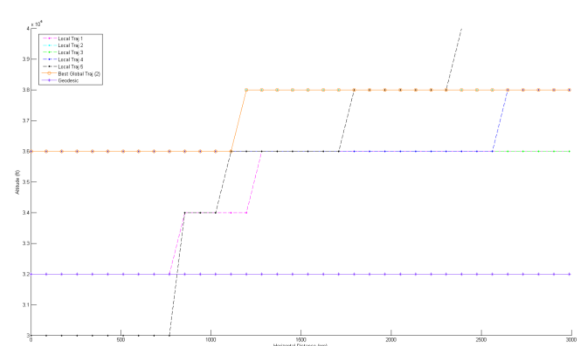
Figure 10. Initial trajectories before PSO

This result confirms that the step-climb is important to reduce fuel burn. However, it is fairly easy achieving good optimization results against geodesic trajectories as step-climbs are known to bring the aircraft to more convenient flight conditions due to fuel burn diminishing weight.

For this reason, it is of interest to compare the optimization against already optimized flights such as the ones available in *flightaware®*.

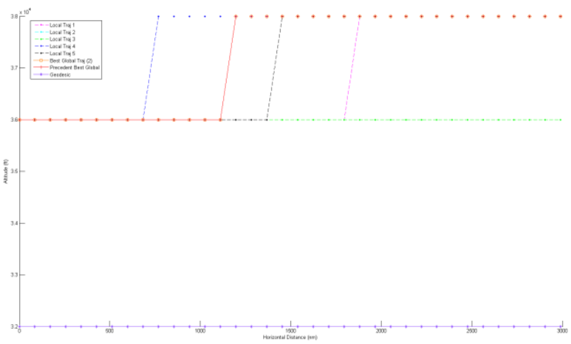

Figure 11. Local trajectories evolution after the required displacements.

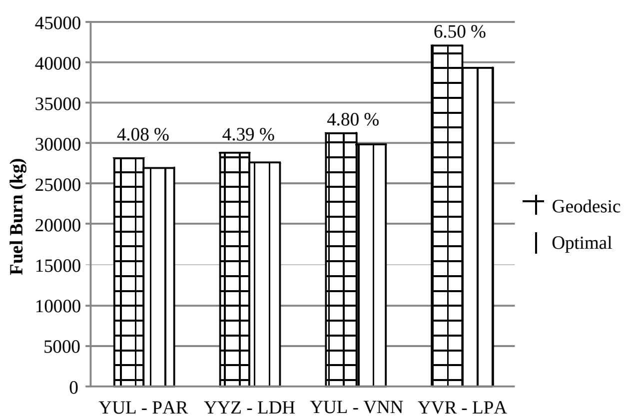


Figure 12. Fuel burn savings between the geodesic trajectory and the optimal trajectory provided by the optimization algorithm

### 4.3. The Optimization Potential Comparing Against Real as Flown Flight

The real flight optimization was carried out by computing the fuel burn required to travel the flight following the same information provided by *flightaware®* such as altitudes, waypoints, and the TAS (converted to Mach). Weather information was obtained from Weather Canada. The same flight was flown using the PSO algorithm in order to obtain the new set of altitudes.

These tests were performed for 4 different real flights: Calgary to Cancun (TSC180), London to Toronto (TSC223), Edmonton to Punta Cana (TSC592), and a second different flight from London to Toronto (TSC723). As seen in Figure 13, for all flights, the algorithm was able to find more fuel efficient reference flight trajectories where up to 3.11% of fuel was saved. This was due a better selection of altitudes which led to achieve efficient performance, and probably to better weather conditions.

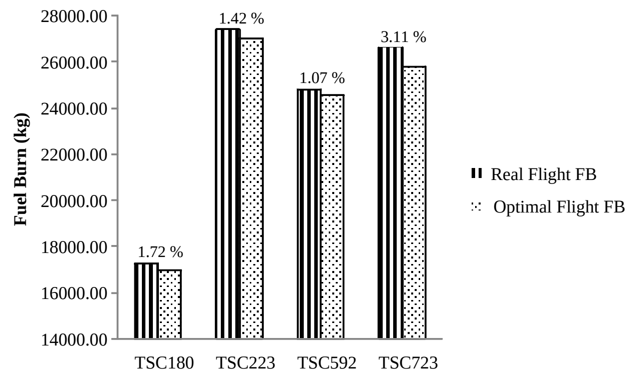


Figure 13. As flown flight fuel burn comparison against the reference flight trajectory delivered by the developed algorithm for four different flights.

## 5.- Conclusions

In this paper, the PSO was tailored to obtain the altitudes per waypoint to deliver the vertical reference trajectory that provides the most economical cost in terms of flight burn.

It was observed that trajectories converged towards the optimal solution. It was as well observed that the algorithm is able to find remarkably good results against simple trajectories.

The algorithm was also able to obtain good results when comparing the fuel burn against a more complex trajectory such as real trajectories obtained from *flightaware®*.

For this reason, it is concluded that the algorithm is able to optimize trajectories. It is now desirable to optimize the trajectory in the lateral dimension, then in 3D in order to reduce even more the fuel requirements for a given flight.

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